Postclosure Care (PCC) Statistics for Leachate Trend Analyses: Kendall's Tau

By Carl E. Burkhead

(11-5-14)

Introduction

This document was prepared for the purpose of providing a reference for the same named power point presentation at the November 20, 2014 SWANA meeting in Manhattan, KS. As the title suggests, the presentation and papers' objective is to discuss the use of statistics, specifically the Kendall's tau procedure, to analyze leachate data (and by inference landfill gas data) to support PCC recommendations.

Background Information and Specific Objectives

Bureau of Waste Management Policy 2014-P2 related to **Reduction and/or Termination of Postclosure Care Activities** (1) makes reference to trend analysis of leachate and landfill gas emissions for MSWLFs that operated after the dates listed in KAR 28-29-100; or MSWLFs that are subject to full Subtitle D requirements. "Each landfill O/O who chooses to demonstrate that one or more PCC activities can be reduced, or terminated must submit a Postclosure Care Reduction and/or Termination Plan to KDHE." The plan should be prepared according to Technical Guidance Document SW-2014-G1 entitled **Preparation of Postclosure Care Reduction and/or Termination Plans** (2). This document provides recommendations for developing a plan that will be used to determine when certain PCC activities at MSWLFs may be reduced and/or terminated. Again, reference is made to trend analysis but with the added requirement that "the confirmation of equilibrium for 15 years (or a trend) prior to the projected PCC activity termination date can be validated by non-parametric statistical analysis." It is this latter requirement and procedure, using leachate emissions data as an example, which is the topic and objective of this document. A secondary objective is to provide reference material for statistics in general and for the proposed methodology.

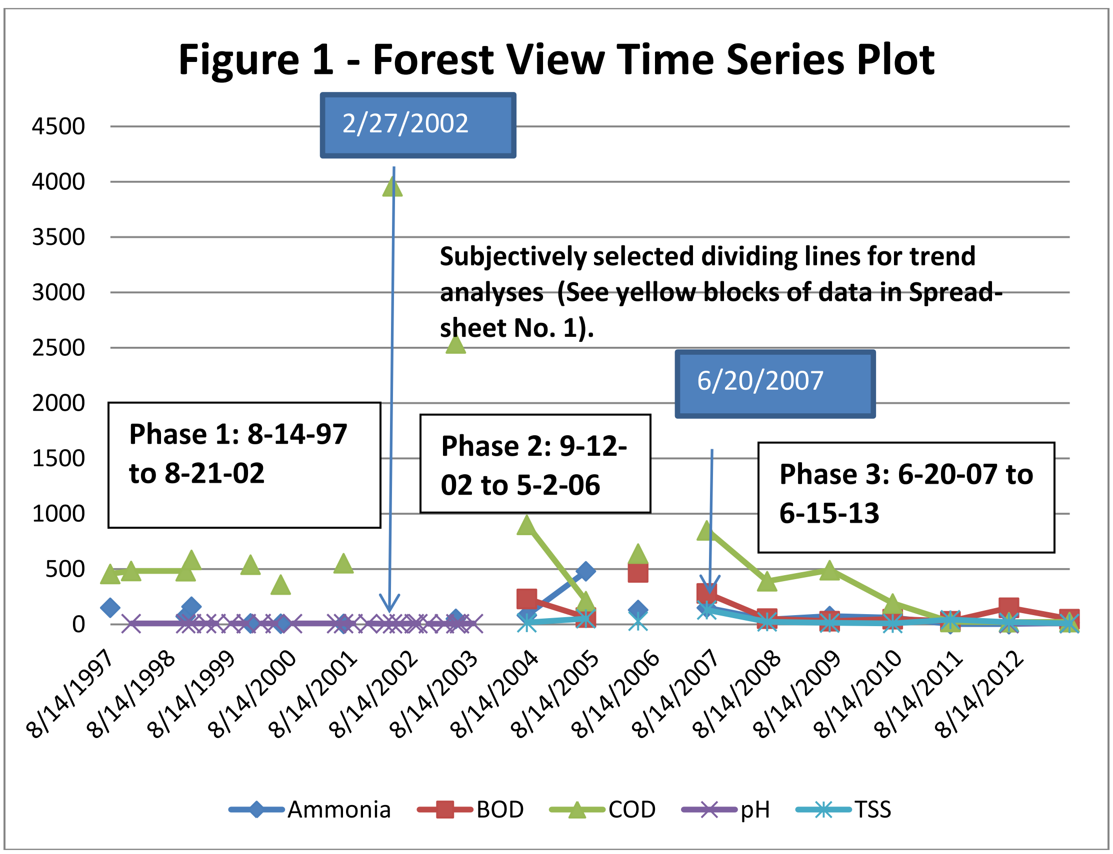
Specific Trend Validation Requirements

The specific "proposed trend methodology to determine when one or more PCC activities may be reduced and/or terminated will involve a minimum of 3 major evaluation periods of at least 5 years each where each period is statistically verifiable. The determination of equilibrium for leachate and landfill gas emissions can be accomplished by quarterly sampling over a 5-year demonstration period, during which there is no statistically significant difference in key parameter values with time. If there is a statistically significant difference in any of the key parameters, a new 5-year demonstration period must begin. After 1 demonstration period in which key parameters are shown to be in equilibrium, reduction of appropriate activities may begin, after approval by KDHE. After 3 consecutive demonstration periods in which key parameters are shown to be in equilibrium, appropriate activities may be terminated, after approval by KDHE." (2)

Selection of MSWLF Leachate Data to Demonstrate the Use of Statistical Analysis to Validate Trend Data

There are eighteen Subtitle D landfills in Kansas; two are closed (Forest View and Wheatland). Of the two closed landfills, Forest View was chosen to demonstrate the use of non-parametric statistics because it has a more complete set of leachate data for statistical analysis. Forest View closed (stopped receiving MSW) on December 29, 2006 and PCC started on December 2, 2010. It is also the first landfill to be subject to the full Subtitle D closure and PCC standards.

Historically, leachate samples at Forest View were collected from a single riser discharging into a force main going to the Kansas City, KS POTW (Kaw Point) starting on August 14, 1997 to September 2013; since that time leachate has been and is being collected from three separate risers, LR-1, LR-2 and LR-3, representing leachate generated from three different phases; Nos. 2, 6 and 13, respectively. None of these latter data are used in this analysis. A summary of the available leachate data for selected parameters representing the stabilization of MSW is given in Figure 1. Three phases, approximately 5 years apart, were arbitrarily selected from a visual comparison of the BOD and COD data; the two key stabilization parameters. Note that the Phase 2 or middle phase data are more limited and more variable.



Selection of Statistical Methodology to Validate Trend Data

The determination of which statistical method to evaluate trend data depends on whether the trend data are normally distributed (parametric) or not (non-parametric). An absolute way to confirm the data type is to plot the data as a histogram as shown in Figure 5-10 taken from Reference 5, page 71 or to plot the data on probability paper to see if it gives a straight line as shown in the Figure 5-9 (ibid, page 70). Another way to "linearize" data is to use a log plot. Figure 5-11 (ibid, page 71) is a transformation of Figure 5-9 data using the logarithm of the concentration values. Figure 5-12 (ibid, page 72) is the histogram of Figure 5-11 data.

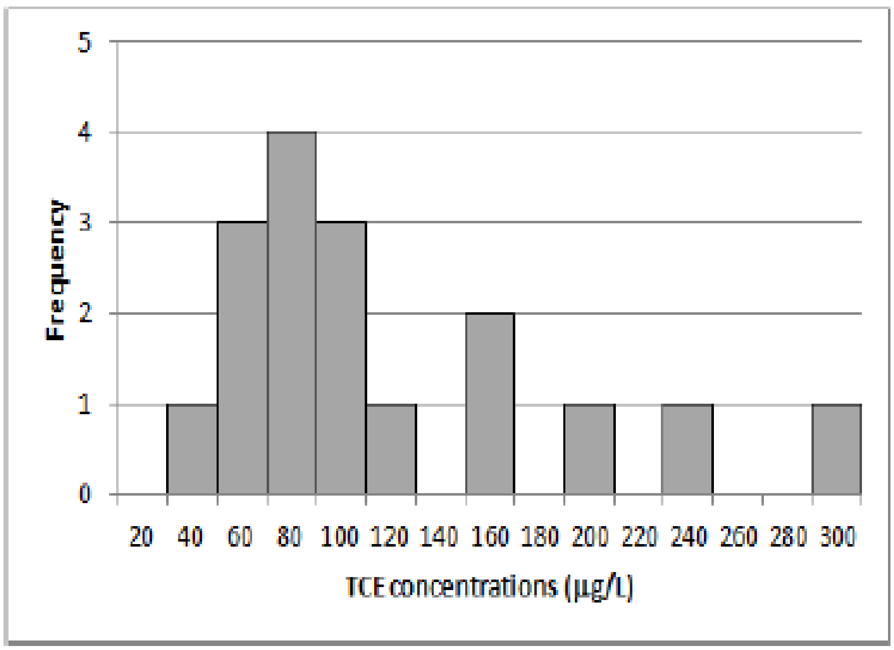


Figure 5-10. Data set as a histogram.

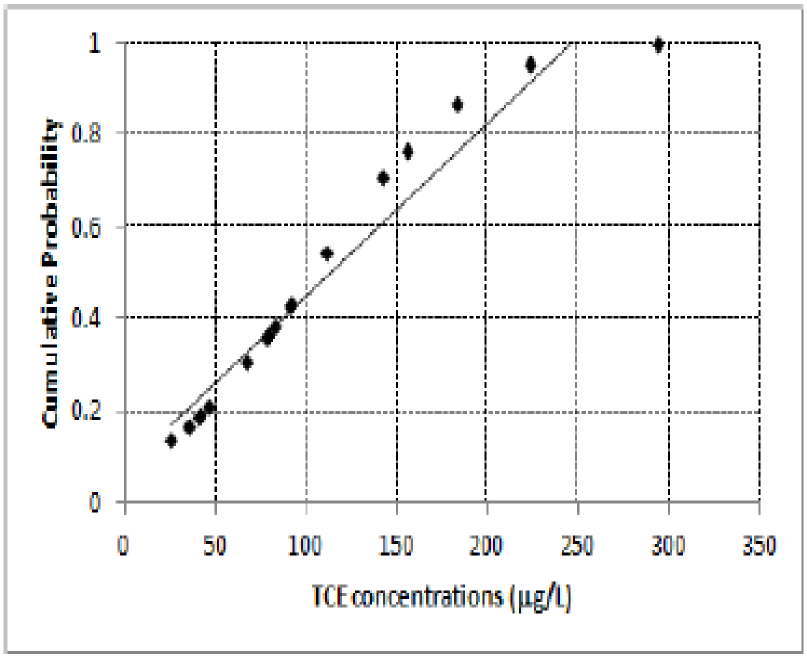


Figure 5-9. Data set as a probability plot.

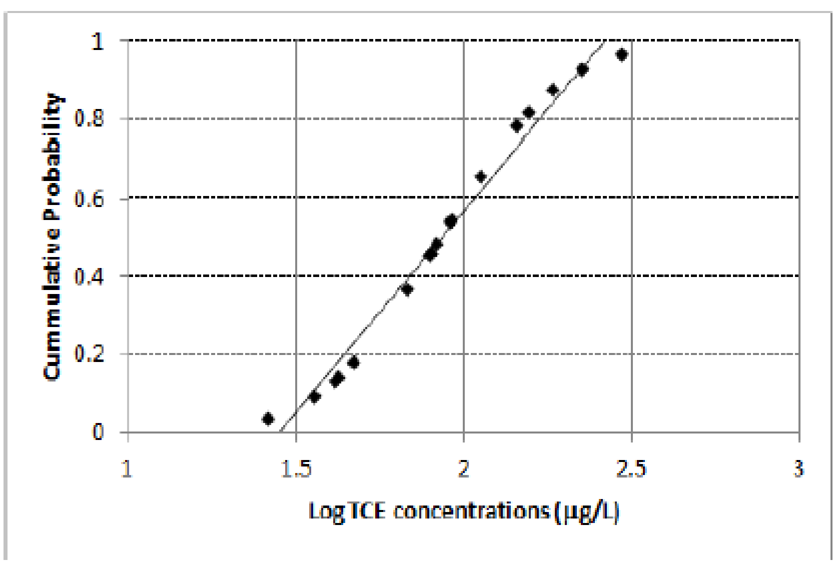


Figure 5-11. Logarithms of data set as a probability plot.

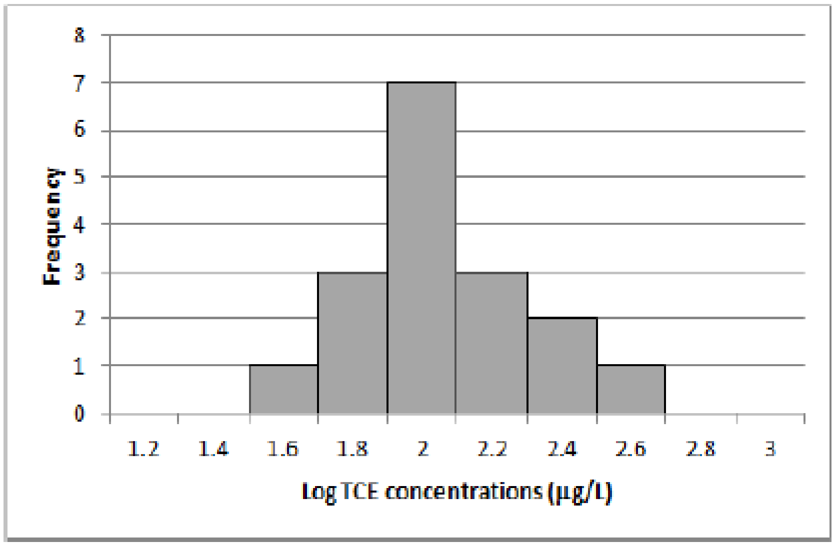


Figure 5-12. Histogram of log-transformed data.

[**Note Announcements:** I will inject notes into this document to share thoughts about statistics in general and to offer insight into the underlying basis of this science although I do not consider myself as a statistician or a skilled user of statistics. However, I believe these comments may be helpful to someone wanting to better appreciate statistics.]

Note 1 – Normal distribution is a unique property of a sample or of the total population. The available data are symmetrical about the mean, median and mode when all are similar in value. Natural or environmental sources of data can be normally distributed although most statisticians say that such data are not normal. This can be determined by plotting a histogram of the data or by plotting the data on probability paper as discussed and shown above.

Note 2 - A series of data over time, e.g., concentrations in a time series plot, may or may not represent the same populations and therefore may not share the same normality. It should be recognized that each parameter has its own unique distribution. If the precision of the test is good, then the normal distribution will be compressed; or if not good, it will be spread out. Either way, perfect normal distribution has a precise shape which can be defined by a formula involving the mean and standard deviation. Statisticians fit a mathematical model to the data and use that model to describe various statistical parameters which help to understand different aspects of the distribution.

There are three reference or guidance documents which provide a thorough review of statistical methods and which have application to this paper's objectives. The references are: the USGS's **Statistical Methods in Water Resources** (3), EPA's **Statistical Analysis of Groundwater Monitoring Data at RCRA Facilities** (4) and ITRC's **Groundwater Statistics and Monitoring Compliance: Statistical Tools for the Project Life Cycle** (5). Chapter 12, **Trend Analysis**, Chapter 17, **Anova, Tolerance Limits, and Trend Tests** and Chapter 5.0, **Statistical**

Tests and Methods, Section 5.5 **Trend Tests** discuss trend statistical methods for the preceding references, respectively. EPA's Chapter 8, **Summary of Recommended Methods** and ITRC's Chapter 5 can be used to select the right statistical method for both parametric and non-parametric distributions. References 6 and 7 include an online statistics tutorial and a useful non-parametric statistics textbook, respectively.

Chapter 8, page 209 of Reference 3 discusses the correlation of two continuous variables such as concentration and time as found in a time series plot; also, "the significance of that association can be tested for, to determine whether the observed pattern differs from what is expected due entirely to chance."

Note 3 – The preceding quote highlights the value of statistics which is based largely on the concepts of probability; which in turn, are based on mathematics. The statement says that the significance of an association can be tested by probability principles, using statistics. This is in sharp contrast to the interpretation of the resultant trend data in terms of the chemistry, microbiology, groundwater hydraulics, other site conditions and operations of the MSWLF regime before and during the placement of the MSW. In other words, there exists supporting or validating evidence provided by statistics to confirm what the direct scientific evidence is demonstrating or suggesting. This note is reinforced by the following quote from Chapter 8, pages 210 to 213 of Reference 3, "Correlation coefficients measure the strength of association between two continuous variables. Of interest is whether one variable generally increases as the second increases, whether it decreases as the second increases, or whether their patterns of variation are totally unrelated. Correlation measures observed co-variation. It does not provide evidence for causal relationship between the two variables. One may cause the other, as precipitation causes runoff. They may also be correlated because both share the same cause, such as two solutes measured at a variety of times or a variety of locations. (Both are caused by variations in the source of the water). Evidence for causation must come from outside the statistical analysis – from the knowledge of the processes involved."

Chapter 8, page 210 of Reference 3 says: "Measures of correlation (here designated in general as p) have the characteristic of being dimensionless and scaled to lie in the range $-1 \le p \le 1$. When there is no correlation between two variables, $p = 0$. When one variable increases as the second increases, p is positive. When they vary in opposite directions, p is negative. The significance of the correlation is evaluated using a hypothesis test where H_0: $p= 0$ versus H_1: $p \neq 0$. When one variable is a measure of time or location, correlation becomes a test for temporal or spatial trend."

Note 4 – A hypothesis test consists of two types (Reference 6, **Choosing a Hypothesis**, Level 1): **Experimental** or **research** (or **alternate** in Reference 4) hypothesis which is the prediction of your theory or "the effect you suspect you will see. This is referred to as H_1" [or H_A in (4)]. The other is the **null** hypothesis which is "the statement that the effect described in the experimental hypothesis does not exist. This is referred to as H_0" [and in (4)].

Referring to Reference 3, Section 8.1.1 Monotonic Versus Linear Correlation, pages 210 to 213: "Data may be correlated in either a linear or nonlinear fashion. When y generally increases or decreases as x increases, the two variables are said to possess a monotonic correlation. This correlation may be nonlinear, with exponential patterns, piecewise linear patterns, or patterns similar to power functions when both variables are non-negative. Figure 8.1 (this and other related figures are shown on page 8) illustrates a nonlinear monotonic association between two variables -- as x increases, y generally increases by an ever-increasing rate. This nonlinearity is evidence that a measure of linear correlation would be inappropriate. The strength of a linear measure will be diluted by nonlinearity, resulting in a lower correlation coefficient and less significance than a linear relationship having the same amount of scatter. Three measures of correlation are in common use -- Kendall's tau, Spearman's rho, and Pearson's r. The first two are based on ranks, and measure all monotonic relationships such as that in Figure 8.1. They are also resistant to effects of outliers. The more commonly-used Pearson's r is a measure of linear correlation (Figure 8.2), one specific type of monotonic correlation. None of the measures will detect nonmonotonic relationships, where the pattern doubles back on itself, like that in Figure 8.3."

Reference 5, pages 133 to 134, compares Spearman's rho, and Pearson's r: "The Spearman rank correlation test is essentially the nonparametric version of the Pearson correlation coefficient test, and provides a measure of the linear association between two variables. Spearman's rank correlation coefficient rho (ρ) is a nonparametric correlation coefficient that can be used to test for monotonic trends." However one can, "Use this test to evaluate stationarity of the mean (the absence of a trend) for parametric data sets, which is a requirement for many statistical methods. A slope differing from zero may indicate the presence of a trend."

Based on the recommendations of Tony Stahl of the Bureau of Water, KDHE, and Mike Johnson of MLJ-LLC Ecosystems Consulting (see **Acknowledgement** section)**,** the Kendall's tau test is the most appropriate statistic methodology for validating leachate trend data. A review of EPA's Chapter 8, **Summary of Recommended Methods**, Tables 8-1, page 8-6 and Section 8-3, pages 8-21 and 8-32 (Note a typo on page 8-1 which says Section 8-2.) cite Mann-Kendall and Theil-Sen as trend tests, respectively. Mann-Kendall hypothesis statements [Null Hypothesis: H_0 is that no discernible linear trend exists in the concentration data over time; and Alternate Hypothesis: H_A is that a non-zero (upward) linear component to the trend does exist] do not seem to apply since we are looking for a zero trend in the leachate data. From Section 8-3, "The Theil-Sen trend line is not a formal hypothesis test but rather an estimation procedure. The algorithm can be modified to formally test whether the true slope is significantly different from zero" statement suggests this method can be used too since we are looking for the confirmation of a zero slope.

Note 5 - No mention is given to the Kendall's tau test in EPA's guidance (4). It is mentioned in the USGS (3) and ITRC (5) guidance documents; and in Conover's 1999 book (7) which is older than any of the comprehensive guidance references where only the Kendall-Mann and Theil-Sen methods are discussed. The Kendall's tau and the Mann-Kendall statistics are essentially the same thing. The Mann-Kendall test calculates a test statistic S. Positive values of S indicate

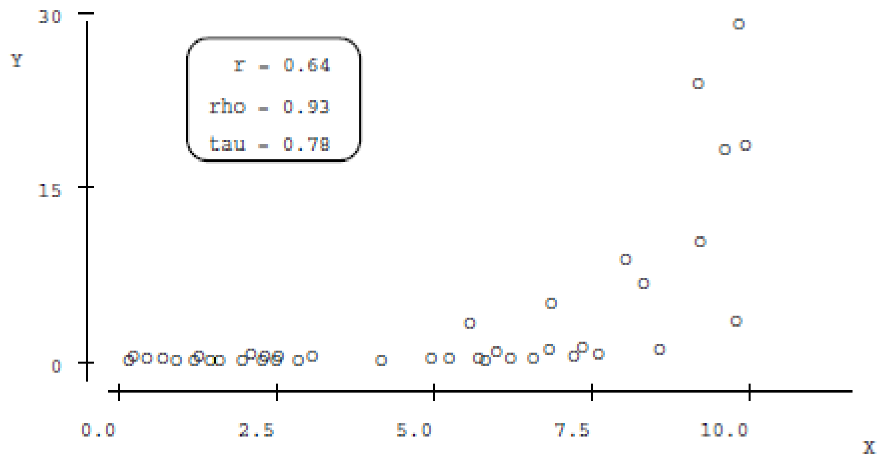


Figure 8.1 Monotonic (nonlinear) correlation between x and y.

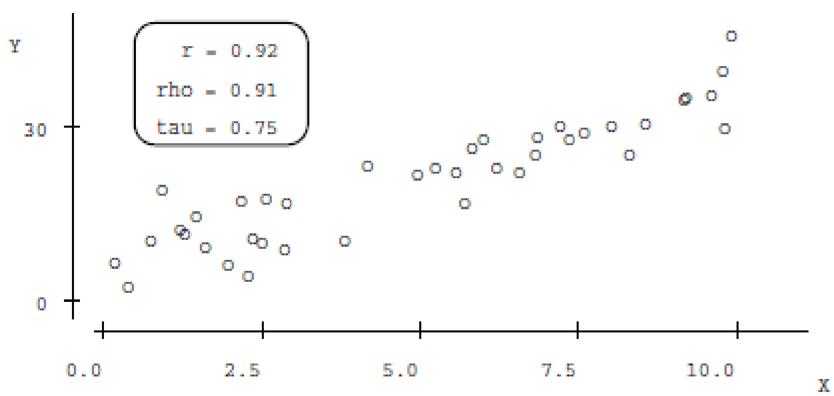


Figure 8.2 Linear correlation between X and Y.

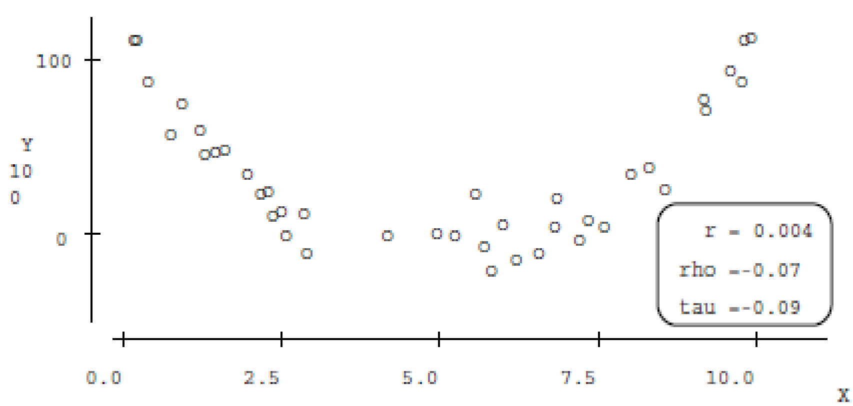


Figure 8.3 Non-monotonic relationship between X and Y.

an upward or increasing trend, while a negative S indicates a downward or decreasing trend in the data. The Kendall's tau statistic is a scaled value of S that lies between -1 and +1. **If these were parametric statistics, the Mann-Kendall test would be a covariance, and Kendall's tau a correlation coefficient. So, both tests essentially measure the same thing, they are just scaled differently.** Also note that two other similar trend tests are for Pearson's coefficient and Spearman's p. ITRC (Reference 5, pages 132 and 133, respectively) says concerning these two trend parameters:

> "The parametric Pearson correlation test provides a measure of the linear association between two continuous variables. To conduct the test, correlation coefficients are calculated for each (x,y) pair, and the values of x and y are subsequently replaced with their ranks. Application of the test results in a correlation coefficient that ranges from -1 to 1. The sign of the coefficient indicates the direction of the relationship (that is, negative values imply an inverse relationship or a decreasing trend), and its absolute value indicates its strength, with larger (absolute) values indicating stronger linear relationships.
>
> "The Spearman rank correlation test is essentially the nonparametric version of the Pearson correlation coefficient test, and provides a measure of the linear association between two variables. Spearman's rank correlation coefficient rho (ρ) is a nonparametric correlation coefficient that can be used to test for monotonic trends. To calculate the correlation coefficient ρ for any pair of variables x and y, each value of x is replaced with its rank R(x) and each corresponding value of y is replaced with its rank R(y). For concentrations sequentially measured over time (such as those, from a monitoring well), the x variable denotes time and R(x) is the sampling event order (R(x) = 1 for the first sampling event). The rank of the smallest concentration measurement is 1 (when it is not tied with other values).
>
> "Spearman's ρ is similar to Pearson's r that is calculated for the paired ranked results (1, R(y1)), (2, R(y2)), … (n, R(yn)) (for instance using Equation 3.5 in Chapter 3.5, Unified Guidance). Like the Pearson's r, Spearman's ρ ranges from -1 to 1 and can be tested to determine whether it is significantly different from zero; a positive value indicates an increasing trend and a negative value indicates a decreasing trend. The absolute value of the coefficient indicates its strength, with larger (absolute) values indicating stronger linear relationships."

Conover (Reference 7, page 323) says when talking about the calculation of two of the above statistics that "The same data were used for both Spearman's p and Kendall's Ƭ in order to compare the two statistics better. It was seen that Spearman's p (p = 0.5900) was a larger number than Kendall's Ƭ (Ƭ = 0.4355). However, the two tests using the two statistics (or their equivalents) produced nearly identical results. Both of the preceding statements hold true in most, but not all, situations. Spearman's p tends to be larger than Kendall's Ƭ, in absolute values. However, as a test of significance there is not strong reason to prefer one over the other, because both will produce nearly identical results in most cases."

Based on the above commentary, none of these other statistical parameters will be discussed in the subsequent sections; however, they can be used as valid statistics given the assumptions that the methods are based on (see Reference 5, Appendix B: **Common Misapplication of Statistics**; and Appendix D: **Software Programs**).

Kendall's tau and Related Parameters

The following sections of Reference 3, pages 212 to 213, are reproduced to describe the procedure to evaluate this non-parametric parameter:

"8.2 Kendall's Tau
Tau (Kendall, 1938 and Kendall, 1975) measures the strength of the monotonic relationship between x and y. Tau is a rank-based procedure and is therefore resistant to the effect of a small number of unusual values. It is well-suited for variables which exhibit skewness around the general relationship.

"Because tau (Ƭ) depends only on the ranks of the data and not the values themselves, it can be implemented even in cases where some of the data are censored, such as concentrations known only as less than the reporting limit. This is an important feature of the test for applications to water resources. See Chapter 13 for more detail on analysis of censored data.

Note 6 – The data used to evaluate the degree of stabilization of the MSW are not censored; hence the preceding advantage is not applicable

"Tau will generally be lower than values of the traditional correlation coefficient r for linear associations of the same strength (Figure 8.2). 'Strong' linear correlations of 0.9 or above correspond to tau values of about 0.7 or above. These lower values do not mean that tau is less sensitive than r, but simply that a different scale of correlation is being used. Tau is easy to compute by hand, resistant to outliers, and measures all monotonic correlations (linear and nonlinear). Its large sample approximation produces p-values very near exact values, even for small sample sizes. As it is a rank correlation method, tau is invariant to monotonic power transformations of one or both variables. For example, Ƭ for the correlation of log(y) versus log(x) will be identical to that of y versus log(x), and of y versus x."

"8.2.1 Computation
Tau is most easily computed by first ordering all data pairs by increasing x. If a positive correlation exists, the y's will increase more often than decrease as x increases. For a negative correlation, the y's will decrease more often than increase. If no correlation exists, the y's will increase and decrease about the same number of times. A two-sided test for correlation will evaluate the following equivalent statements for the null hypothesis H_0, as compared to the alternate hypothesis H_1:
H_0:

a) no correlation exists between x and y (Ƭ= 0), or

b) x and y are independent, or

c) the distribution of y does not depend on x, or

d) Prob ($y_i < y_j$ for $i < j$) = 1/2.

H_1:

a) x and y are correlated ($\tau \neq 0$), or

b) x and y are dependent, or

c) the distribution of y (percentiles, etc.) depends on x, or

d) Prob ($y_i < y_j$ for $i < j$) ≠ 1/2.

"The test statistic S measures the monotonic dependence of y on x. Kendall's S is calculated by subtracting the number of "discordant pairs" M, the number of (x,y) pairs where y decreases as x increases, from the number of "concordant pairs" P, the number of (x,y) pairs where y increases with increasing x:

$S = P - M$ [8.1]

Where P = "number of pluses", the number of times the y's increase as the x's increase, or the number of $y_i < y_j$ for all $i < j$,

M = "number of minuses," the number of times the y's decrease as the x's increase, or the number of $y_i > y_j$ for $i < j$.

for all i = 1,....(n − 1) and j = (i+1),.....n.

"Note that there are n(n−1)/2 possible comparisons to be made among the n data pairs. If all y values increased along with the x values, S = n(n−1)/2. In this situation, the correlation coefficient τ should equal +1. When all y values decrease with increasing x, S = −n(n−1)/2 and τ should equal −1. Therefore dividing S by n(n−1)/2 will give a value always falling between −1 and +1. This then is the definition of τ, measuring the strength of the monotonic association between two variables:

Kendall's tau correlation coefficient

$$\tau = S/n(n-1)/2 \qquad [8.2]$$

To test for significance of τ, S is compared to what would be expected when the null hypothesis is true. If it is further from 0 than expected, H_0 is rejected. For $n \leq 10$ an exact test should be computed. The table of exact critical values is found in table B8 of the Appendix."

Note 7 - Reference 5, page 91, refers to the above test statistic, S, as the Mann Kendall statistic with the following discussion: "The Mann Kendall statistic (S) is calculated through pair-wise comparisons of each data point with all preceding data points, and determining the number of increases, decreases, and ties. Pairs of nondetects below the reporting limit are "ties" that do not increase or decrease the value of S. A positive value for S implies an upward or increasing temporal trend, whereas a negative value implies a downward or decreasing trend. **A value of S near zero suggests there is no significant upward or downward trend** (author's emphasis)**.** The magnitude of S measures the "strength" of the trend. A statistically significant trend is reported if the absolute value of S is greater than the "critical value" of S (obtained from a table)," as mentioned above.

Note 8 - According to the last sentence in the above quote and the one from Reference 3, page 213, if the numbers of data pairs are equal to or less than 10, then an exact test should be computed using the critical values in Table B8 in the Appendix of Reference 3. This calculation is not shown since there will be normally more than 10 leachate data collected over a five year period.

Kendall's tau

In order to better understand the basis for the Kendall's tau method and to demonstrate the methodology, different example calculations are presented. The first two are taken from Helsel (8) and the third is based on Forest View data.

Table 1 is a copy of a spreadsheet used to calculate Kendall's tau from nine data pairs taken from Helsel (Reference 8, page 190). The dependent parameter, Y, represents dissolved iron concentration values with six censored (non-detect) values. Each concentration value allows a determination of discordant values [Y decreases with X (time) increases], e.g., between Y = 20 and Y = >10, there is one M value designated with a minus. Between Y = 20 and the next two Y's, there is one M value (or minus), and one which is neither concordant nor discordant [designated '0" which is called a "tie" and will be discussed in the next section entitled **Kendall's tau b** or the **gamma coefficient** as discussed in Reference 7, pages 189*ff*]; and so on, giving total pairs, 13 discordants and 23 ties. Therefore, S = P – M or 0 – 13 = -13 and Ƭ = *S/n*(*n* −1)/2 = 13/9(9-1)/2 = 13(2)/9(8) = -0.36.

Table 1 – Determination of Kendall's tau as per Helsel (8)

Y Kendall tau Pairs Determination: Hensel Example on pages 190 to 191 of his book.*										
X (Sum of Pairs = n = 9)	**Y**	**(Yi - Yj)$_1$**	**(Yi - Yj)$_2$**	**(Yi - Yj)$_3$**	**(Yi - Yj)$_4$**	**(Yi - Yj)$_5$**	**(Yi - Yj)$_6$**	**(Yi - Yj)$_7$**	**(Yi - Yj)$_8$**	
1977	20									
1978	<10	–								
1979	<10	–	0							
1980	<10	–	0	0						
1981	<10	–	0	0	0					
1982	7	–	0	0	0	0				
1983	3	–	0	0	0	0	–			
1984	<3	–	0	0	0	0	–	–		
1985	<3	–	0	0	0	0	–	–	0	
Pairs Determinations:		8	7	6	5	4	3	2	1	36
Sum of Discordants = Nd		8					3	2		13
			7	6	5	4			1	23
There are no concordants (Nc) since Y doesn't increase as X does.										
***Nondetects and data analysis: statistics for censored environmental data.**										

Note 9 - According to Hensel, **Kendall's tau** is a correlation coefficient computed by "comparing all pairs of observations, counting the number of positive slopes minus the number of negative slopes, and dividing by the total number of pairs of observations." The equation is: **Kendall's**

tau = (Nc - Nd)/n(n-1)/2 or 2(Nc-Nd)/n(n-1) which is the same as Equation 8.2 above. The numerator (without the 2) would represent the difference between the number of positive and negative slopes which as shown above is: (0 - 13) or -13, i.e., all the slopes are negative. Note that the actual slopes of the pairs are not determined; only the fact that they are plus or minus. The denominator is 9(9 -1)/2 or 36, which is a mathematical calculation of the total number of pairs as show above. This gives a **Kendall's tau** of -13/36 or - 0.36. This suggests that **Kendall's tau** is a "pseudo" mean value of the number of slopes. If there is complete concordance or discordance in the data, then **Kendall's tau = + 1.0 or -1.0**, respectively; or, a perfect correlation is assumed even if the data are non-linear. Another way of saying this is to say that a trend exists for linear and non-linear data. If a **Kendall's tau = 0**, then there is a zero slope, which is still a trend, or a equilibrium value as described and desired in References 1 and 2.

In summary, according to Reference 5, page 91: "The nonparametric correlation coefficient Kendall's tau (τ) can be calculated to evaluate the nonparametric correlation between two data series. It is essentially a scaled measure of S; $\tau = S/[n(n-1)/2]$, where n denotes the number of concentration measurements. Therefore, a statistical trend is equivalently demonstrated when τ is significantly different from zero. However, it is more convenient to evaluate trends using Kendall's tau, because like the parametric linear correlation coefficient r, τ ranges from -1 to 1. A trend is "strong" if the absolute value of τ is near one." This latter statement suggests that the T value of -0.36 calculated for Helsel's data is not too strong (< 0.7, see page 9 comments); primarily due to the number of nondetects or ties.

Kendall's tau b

Helsel gives a way to calculate Kendall's tau if there are ties involved. The resultant value is called **Kendall's tau b** which can be calculated from the following equation:

$$\text{Kendall's tau b} = \frac{Nc - Nd}{([N(N-1)/2] - \#\text{ of ties}_x) - [N(N-1)/2] - \#\text{ of ties}_y)^{1/2}}$$

Where: N = n above, Nc = P above and Nd = M above in Equations 8.1 and 8.2.

Using the data from Table 1:

$$\text{Kendall's tau b} = \frac{0 - 13}{([9(9-1)/2] - 0) - [9(9-1)/2] - 23)^{1/2}} = -0.60$$

Again, as per the previous summary concerning Kendall's tau, the value is not too strong because it is < 0.7.

Note 10 – Conover (Reference 7, pages 319 to 321) gives an alternate procedure for calculating Kendall's tau if ties are involved but he doesn't call the result Kendall's tau b; it's the same as the basic Kendall's tau calculation except the ties are included in the determination of Kendall's tau. This method is not discussed since Conover's example is only for calculating Kendall's tau with ties (not without) and since his book is not as accessible as Helsel's reference.

Forest View

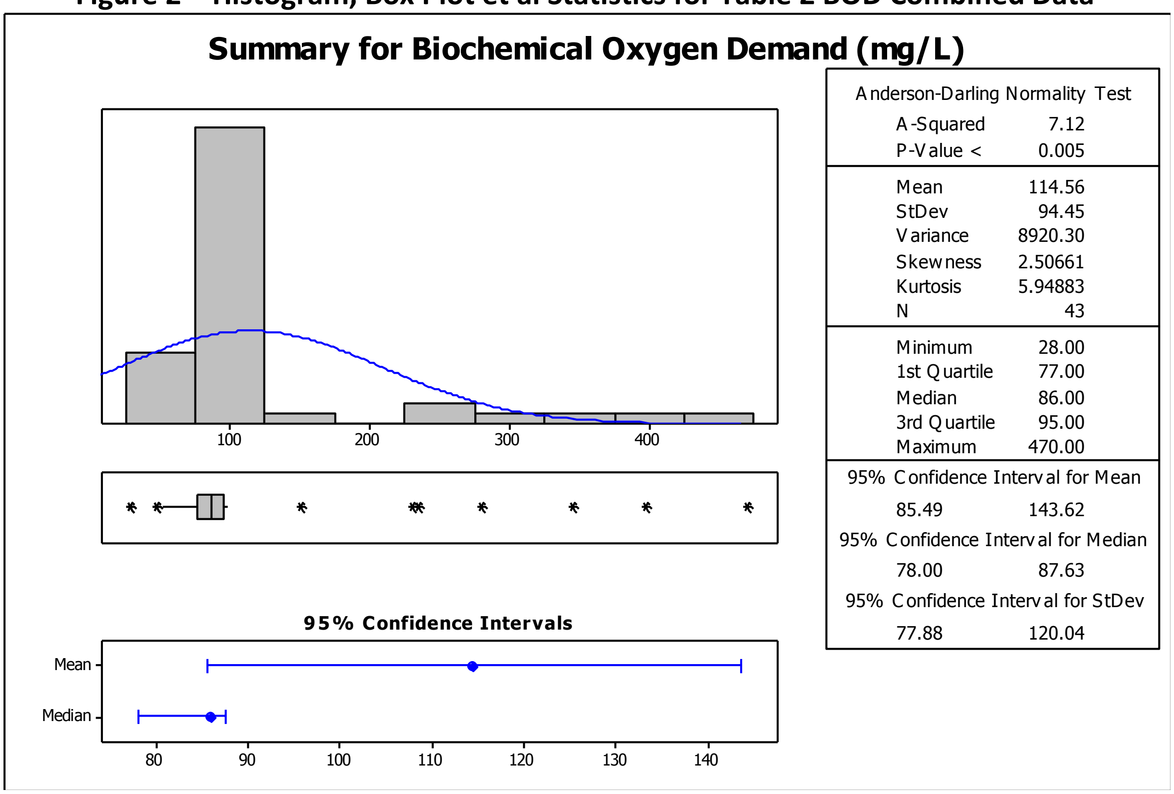
Forest View data are given in Table 2 for the purpose of calculating the preceding various statistics for key stabilization parameters. Table 2 data are divided into three phase as show in Figure 1. The black data are actual data and the red are assumed, hypothetical data for demonstration purposes. **Table 2 – Forest View Postclosure Data** are given on the following page.

Normality Tests

Tony Stahl provided the results shown in this section. They were derived from the use of **Minitab** (9) software to check the distribution of Table 2 data and to transform it (if found to be non-parametric); however, only BOD examples are given. Figures 2 and 3 compare the resulting statistics for the combined data and the actual data, respectively. Figure 4 shows a probability plot and resulting statistics for the combined data.

Note 11 - Minitab's approximate cost is $1,395 for a single user license and $2 940 for 5 multi-user license. Operating system needs include XP, Vista, Windows 7 or 8 (5, page 278).

Figure 2 – Histogram, Box Plot et al Statistics for Table 2 BOD Combined Data



		8/14/1997	12/18/1997	11/12/1998	12/17/1998	3/18/1999	5/14/1999	6/3/1999
NH_3	mg/L	***150***	77	***72***	***160***	86	75	45
BOD	mg/L	87	77	76	86	95	99	87
COD	mg/L	***456***	***483***	***481***	***583***	555	513	605
pH	S.U.	6.7	***6.7***	***6.6***	***6.7***	***6.8***	***7.2***	6.8
TSS	mg/L	45	57	43	55	59	62	54
		8/12/1999	**11/12/1999**	**12/8/1999**	**2/10/2000**	**5/4/2000**	**6/7/2000**	**8/17/2000**
NH_3	mg/L	33	18	***8.6***	10	7	***5.9***	4
BOD	mg/L	87	90	86	77	87	93	77
COD	mg/L	565	585	***540***	445	353	***360***	515
pH	S.U.	***6.6***	***6.6***	6.7	***7.3***	***6.8***	7	***6.5***
TSS	mg/L	58	74	68	73	52	59	67
		5/10/2001	**6/21/2001**	**8/17/2001**	**10/24/2001**	**11/15/2001**	**2/27/2002**	**4/12/2002**
NH_3	mg/L	5	***4.3***	5	6.6	5.9	8.3	7.8
BOD	mg/L	87	78	95	87	77	84	345
COD	mg/L	453	***553***	600	585	585	630	***3960***
pH	S.U.	***6.74***	***6.48***	***6.51***	6.5	***6.17***	***5.96***	***5.28***
TSS	mg/L	72	63	88	66	59	66	69
		5/22/2002	**8/21/2002**	**9/12/2002**	**10/7/2002**	**11/14/2002**	**2/19/2003**	**4/29/2003**
NH_3	mg/L	13	15	24	20	32	45	***50***
BOD	mg/L	86	77	85	76	89	99	235
COD	mg/L	575	606	585	665	685	590	***2540***
pH	S.U.	***5.99***	***6.09***	***6.33***	6.2	***5.86***	***5.68***	***5.65***
TSS	mg/L	73	75	67	72	66	66	70
		5/21/2003	**8/13/2003**	**10/21/2003**	**6/30/2004**	**6/21/2005**	**6/21/2005**	**5/2/2006**
NH_3	mg/L	48	63	72	**82**	**480**	95	**130**
BOD	mg/L	83	69	99	**230**	**62**	78	**470**
COD	mg/L	650	700	685	**900**	**210**	565	**640**
pH	S.U.	**6.05**	**6.87**	6.8	6.6	6	6.6	6.8
TSS	mg/L	55	45	35	**15**	**51**	30	**29**
		5/2/2006	**6/20/2007**	**6/18/2008**	**6/30/2009**	**7/15/2010**	**6/28/2011**	**6/15/2012**
NH_3	mg/L	122	**150**	**43**	**75**	**59.9**	**2.36**	**1.8**
BOD	mg/L	398	**280**	**52**	**29**	**47**	**28**	**150**
COD	mg/L	740	**850**	**390**	**490**	**189**	**22**	**20**
pH	S.U.	6.9	6.7	6.9	6.9	6.7	6.6	**6.58**
TSS	mg/L	25	**130**	**23**	**12**	**8**	**42**	**18**
		6/15/2013						
NH_3	mg/L	**14.6**						
BOD	mg/L	**47**						
COD	mg/L	**21**						
pH	S.U.	**6.26**						
TSS	mg/L	**8**						

The BOD data are not normally distributed as shown in the histogram and probability plots (Figures 2 and 4). The box-whisker plot in Figure 2 shows that there are several outliers which is not surprising given the suspected effect of improper leachate sampling (sampling in the discharge to the POTW piping rather than at the sump). The other statistics are not discussed but they tell a story about the non-parametric nature of the BOD data, e.g., the Anderson-Darling values, since they are less than 0.005, rejects the null hypothesis that the data follows normal distribution.

Figure 3 includes only the actual Forest View data, i.e., the data without the hypothetical values shown in Table 2. In this case, the data are normally distributed according to the Anderson-Darling test for normality since the p value is greater than 0.005. Also, there are no outliers as shown in the box plot.

Figure 3 - Histogram, Box Plot et al Statistics for Table 2 BOD Actual Data

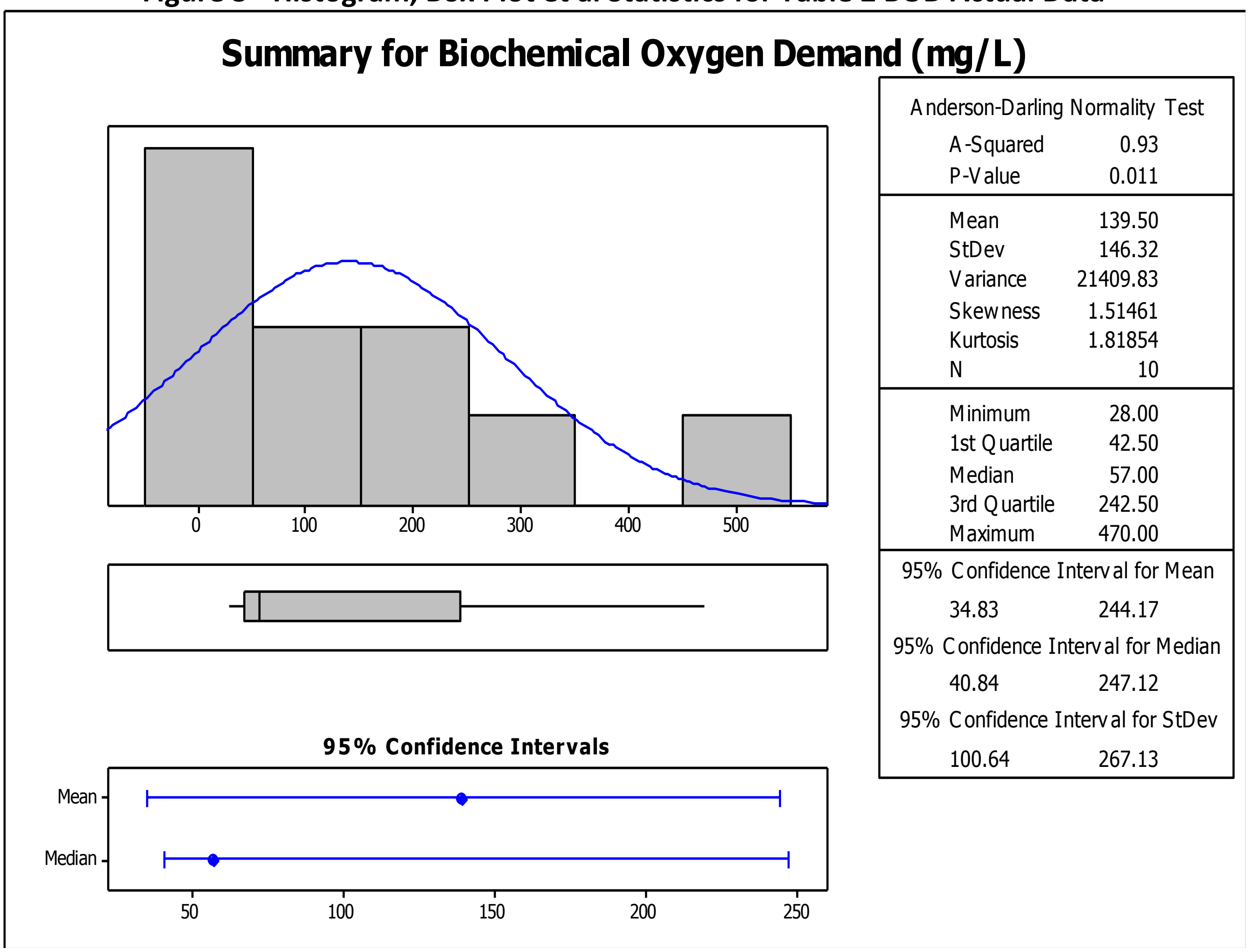
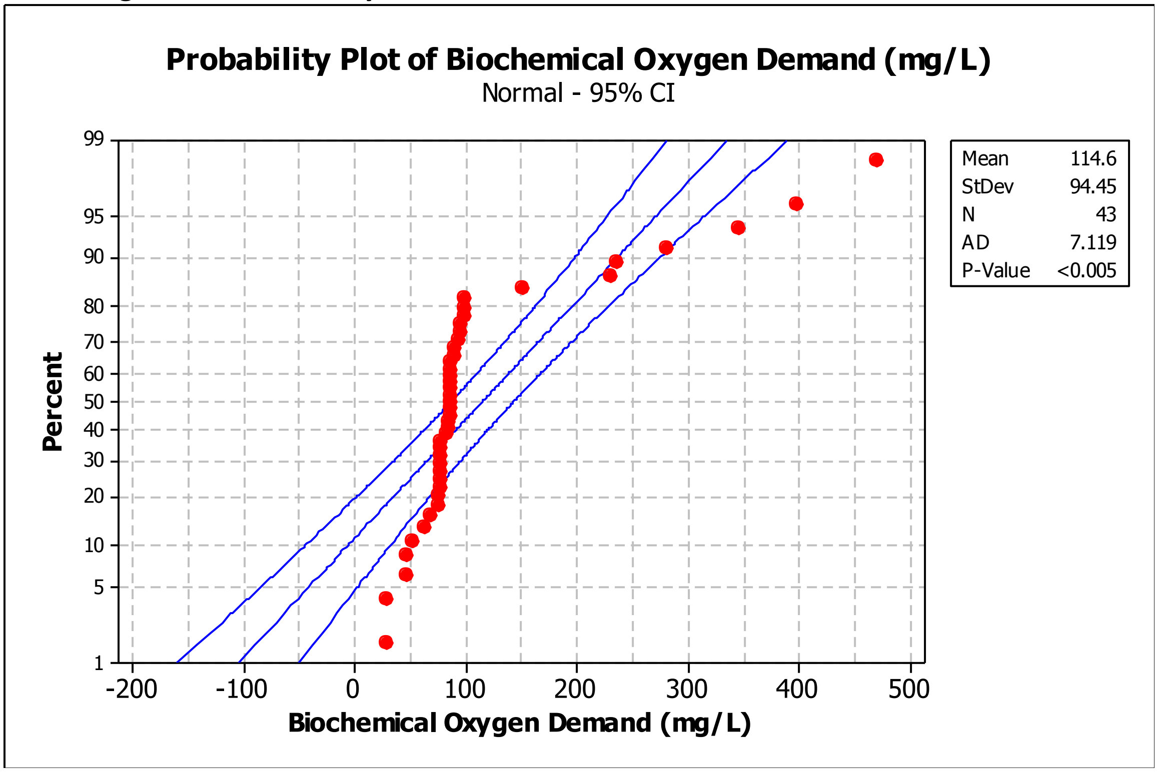


Figure 4 – Probability Plot et al Statistics for Table 2 BOD Combined Data



Kendall's Tau Determinations

Table 3 was constructed to allow Kendall's Tau concordants to be identified for a Phase 1 parameter (fill in the parameter type using the blank space given in the title) and a given number of measurements. It can be a simple hand-filled form or a spreadsheet compilation. Either way, the increases are given a **+**, the decreases a **–** and ties are given a **0** designation. The **Calculation** part of the table is for calculating Kendall's tau longhand.

Note 12 – If the above is done using a spreadsheet (I used **Excel** in Microsoft 7 Office software), the best way to enter **+**, **-** or **0** is to hit the **Enter** key after the selection is made. This will prevent the cell identifier to replace your selected symbol. Also, hitting **Enter** takes you to the cell below the one where you just entered your decision. Or, you can go across using the **Tab** (not the **Arrow**) key, but you must hit **Enter** when you go to the next row; otherwise you will replace the decision with the cell identifier. Sometimes when you enter a decision, you will get a "small triangle" in the upper left part of the cell. To rid these marks, you have to left or right click on the cell and choose "ignore error" in the drop-down icon. I am not sure why they are produced but they will affect the summation part of the table even when they are removed.

Table 3 - Phase 1 Form for Forest View _______Data to Determine Kendall Tau

X (Date)	Y (mg/L)	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	Δ_{11}	Δ_{12}	Δ_{13}	Δ_{14}	Δ_{15}	Δ_{16}	Δ_{17}	Δ_{18}	Δ_{19}	Δ_{20}	Sum
1997																						
1997																						
1998																						
1998																						
1999																						
1999																						
1999																						
1999																						
1999																						
1999																						
2000																						
2000																						
2000																						
2000																						
2001																						
2001																						
2001																						
2001																						
2001																						
2002																						
Nc																						
Nd																						
Calculation:																						

Table 4 is a completed Table 3 for the BOD values given in Table 2. The values for Nc and Nd are used to calculate Kendall's tau using Helsel's version of Equation 8.2 as follows:

Ƭ = 2(Nc- Nd)/n(n – 1) = 2(80 – 87)/20(20 – 1) = 2(-7)/20(19) = - 0.037

A value of – 0.037 based on the discussion in **Note 9** indicates that there are no upward or downward trends, but the value is not statistically different from zero which means that there is no statistically significant trend to the data; upward or downward, or a condition which indicates equilibrium has been established. Similar results can be determined for the other stabilization parameters. The resultant Kendall's tau values (and other statistics and trend conclusions) for ammonia, BOD, COD, pH and TSS are given in Table 5 for each of the preselected phases shown in Table 2. It is apparent that the Kendall's tau BOD values based on Table 4 (-0.037) is different than Table 5 (-0.050) but both results indicate no trend which is the same for the later phase conclusions. This would satisfy the BWM criteria for activity reduction in terms of measuring this parameter. The other parameters do not show "no trend" for the required three five year periods; however, it should be remembered that the leachate values were not collected according to the protocol defined in Reference 11.

Table 4 - Phase 1 Form for Forest View _BOD_Data to Determine Kendall's Tau

X Date	Y mg/L	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	Δ_{11}	Δ_{12}	Δ_{13}	Δ_{14}	Δ_{15}	Δ_{16}	Δ_{17}	Δ_{18}	Δ_{19}	Sum
1997	87																				
1997	77	-																			
1998	76	-	-																		
1998	86	-	+	+																	
1999	95	+	+	+	+																
1999	99	+	+	+	+	+															
1999	87	0	+	+	+	-	-														
1999	87	0	+	+	+	-	-	0													
1999	90	+	+	+	+	-	-	+	+												
1999	86	-	+	+	0	-	-	-	-	-											
2000	77	-	0	+	-	-	-	-	-	-	-										
2000	87	0	+	+	+	-	-	0	0	-	+	+									
2000	93	+	+	+	+	-	-	+	+	+	+	+	+								
2000	77	-	0	+	-	-	-	-	-	-	-	0	-	-							
2001	87	0	+	+	+	-	-	0	0	-	+	+	0	-	+						
2001	78	-	+	+	-	-	-	-	-	-	-	+	-	-	+	-					
2001	95	+	+	+	+	0	-	+	+	+	+	+	+	+	+	+	+				
2001	87	0	+	+	+	-	-	0	0	-	+	+	0	-	+	0	+	-			
2001	77	-	0	+	-	-	-	-	-	-	-	0	-	-	0	-	-	-	-		
2002	84	-	+	+	-	-	-	-	-	-	-	+	-	-	+	-	+	-	-	+	
Nc		5	14	17	10	1	0	3	3	2	5	7	2	1	5	1	3	0	0	1	80
Nd		9	1	0	5	13	14	6	6	9	5	0	4	6	0	3	1	3	2	0	87

Note 12 - These trends can be view by the manipulation and inspection of Figure 1 when the other parameters are removed from the figure by right clicking a particular parameter datum and selecting "Delete." They can be added back by clicking on the "reverse" arrow.

Note 13 - It should be remembered that a variation of the Kendall's tau parameter is the **Kendall's tau b** (see previous section) parameter which takes into account the "ties" found in the data. For the above BOD examples there were 23 ties out of 190 total comparisons. No attempt was made to make this calculation; but, it is speculated, based on the example given by Conover, that the interpretation of the data by either method doesn't change the resulting conclusion.

Table 5 – Minitab Statistics for Forest View (Source: Tony Stahl, KDHE)

Constituent	Start Date	End Date	N	*p* value	Slope	*tau*	Trend
Ammonia	8/14/1997	8/21/2002	23	0.008	-16.818	-0.500	decreasing
Biochemical oxygen demand	8/14/1997	8/21/2002	23	0.851	0.000	-0.050	no trend
Chemical oxygen demand	8/14/1997	8/21/2002	23	0.010	26.294	0.483	increasing
pH	8/14/1997	8/21/2002	23	0.001	-0.109	-0.617	decreasing
Total suspended solids	8/14/1997	8/21/2002	23	0.014	5.284	0.467	increasing
Ammonia	9/12/2002	5/2/2006	13	0.000	30.793	0.872	increasing
Biochemical oxygen demand	9/12/2002	5/2/2006	13	0.326	34.783	0.218	no trend
Chemical oxygen demand	9/12/2002	5/2/2006	13	0.902	13.062	0.038	no trend
pH	9/12/2002	5/2/2006	13	0.124	0.166	0.333	no trend
Total suspended solids	9/12/2002	5/2/2006	13	0.001	-13.000	-0.705	decreasing
Ammonia	6/20/2007	6/15/2013	7	0.072	-15.525	-0.619	no trend
Biochemical oxygen demand	6/20/2007	6/15/2013	7	0.448	-2.413	-0.286	no trend
Chemical oxygen demand	6/20/2007	6/15/2013	7	0.017	-121.616	-0.810	decreasing
pH	6/20/2007	6/15/2013	7	0.031	-0.097	-0.714	decreasing
Total suspended solids	6/20/2007	6/15/2003	7	0.172	-7.238	-0.476	no trend

Hypothetical Trend Calculations

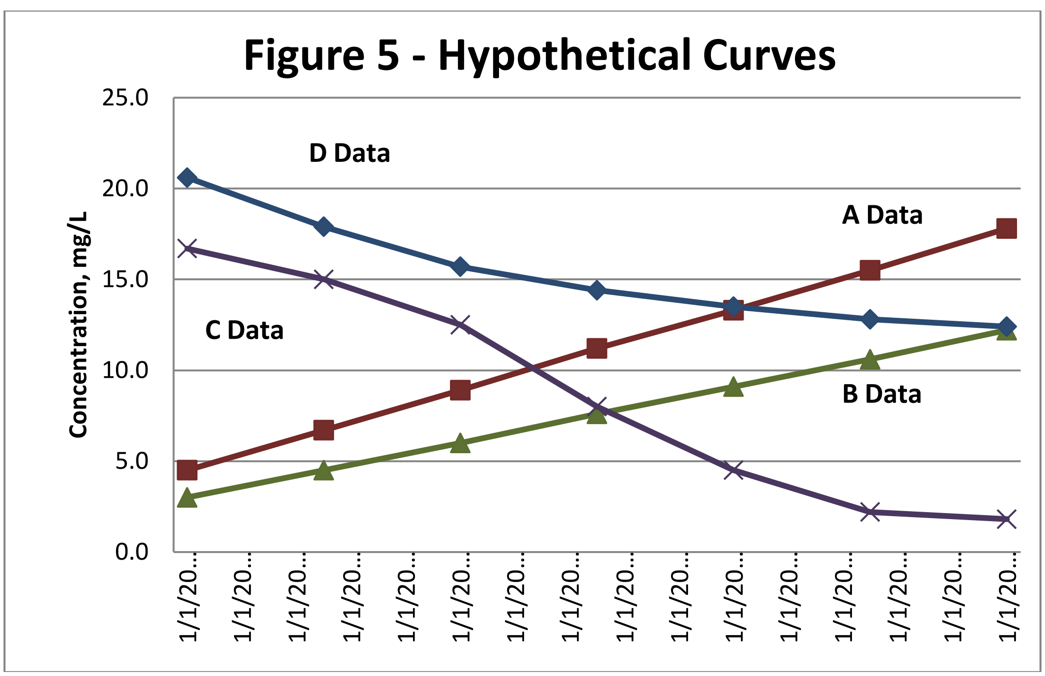
It was decided that assumed curves should be drawn to demonstrate their resultant Kendall's tau values. Figure 5 was drawn using the assumed data given in Table 6. Kendall's tau's were calculated as: **(Nc-Nd)/n(n-1)/2** = 2(21 - 0)/7(7 - 1) = 42/42 = 1.00 for Data Set A and B; and 2(0 -21)/7(7 – 1) = - 42/42 = - 1.00 for Data Set C and D. These results indicate that there is a perfect increasing and decreasing trend for Data Sets A and B; and C and D, respectively, as per **Note 9**. The values don't change unless there is a change in slope from positive to negative or negative to positive.

Note 14 - It is clear from an inspection of Table 6 that all of the comparisons are the same in each data set. This, when compared with the data in Table 4, shows the basis for the Kendall's tau method. It determines all the possible slopes between data points and weighs the totals (+ or -) to compute a type of trend correlation coefficient. The hypothetical curves in Figure 5 show a "perfect" trend, either increasing or decreasing. Notice that the conclusions about the curves do not depend on the magnitude of the slopes; just the fact that they are, as a group, increasing or decreasing.

Table 6 – Data to Demonstrate Kendall's tau Pairs Determinations for Hypothetical Trends as shown in Figure 5

Data	X (Date)	Y (mg/L)	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Sum	Data	X (Date)	Y (mg/L)	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Sum
A	2010	4.5								**C**	2010	16.7							
	2015	6.7	+								2015	15.0	-						
	2020	8.9	+	+							2020	12.5	-	-					
	2025	11.2	+	+	+						2025	8.0	-	-	-				
	2030	13.3	+	+	+	+					2030	4.5	-	-	-	-			
	2035	15.5	+	+	+	+	+				2035	2.2	-	-	-	-	-		
	2040	17.8	+	+	+	+	+	+			2040	1.8	-	-	-	-	-	-	
	Nc		6	5	4	3	2	1	21		Nc		6	5	4	3	2	1	21
	Nd		0	0	0	0	0	0	0		Nd		0	0	0	0	0	0	0
Data	**X (Date)**	**Y (mg/L)**	**Δ_1**	**Δ_2**	**Δ_3**	**Δ_4**	**Δ_5**	**Δ_6**	**Sum**	**Data**	**X (Date)**	**Y (mg/L)**	**Δ_1**	**Δ_2**	**Δ_3**	**Δ_4**	**Δ_5**	**Δ_6**	**Sum**
B	2010	3.0								**D**	2010	20.6							
	2015	4.5	+								2015	17.9	-						
	2020	6.0	+	+							2020	15.5	-	-					
	2025	7.6	+	+	+						2025	14.4	-	-	-				
	2030	9.1	+	+	+	+					2030	13.5	-	-	-	-			
	2035	10.6	+	+	+	+	+				2035	12.8	-	-	-	-	-		
	2040	12.2	+	+	+	+	+	+			2040	12.4	-	-	-	-	-	-	
	Nc		6	5	4	3	2	1	21		Nc		6	5	4	3	2	1	21
	Nd		0	0	0	0	0	0	0		Nd		0	0	0	0	0	0	0

Kendall's tau = (Nc-Nd)/n(n-1)/2



Statistic Analysis Procedure for Validating Emission Data

Subtitle D O/O have the option of hiring a statistician to evaluate their emission data or they can perform all or part of the evaluation themselves. This recommended procedure is for the latter choice where the statistical expertise of the Subtitle D staff is minimal; however, either way, it must be remembered that “good data” give “good results” and the use of statistics to validate PCC plans is secondary to supporting proposed plans with good scientific evidence based on a knowledge of the various processes that produced the data (see **Note 3**). The ways to obtain good data have been outlined in the indicated guidance documents located on the following BWM websites for : LFG (10) and leachate (11) located at http://www.kdheks.gov/waste/techguide/SW-2014-G2.pdf and http://www.kdheks.gov/waste/techguide/SW-2013-G3.pdf, respectively.

The following steps begin with the point where the best data possible (within available economic constraints) have been collected. These data include (not only laboratory results) but all other sources of information as discussed in the **Preparation of Post-Closure Care Reduction and/or Termination Plans (SW-2014-G1)** located at: http://www.kdheks.gov/waste/techguide/SW-2013-G3.pdf. Also, it should be recognized that a successful statistical analysis approach must planned as part of the sampling program, e.g., quarterly sampling is recommended to ensure that phase evaluations have enough data to be completed; in general, the more samples that are taken, the better the overall results. Most statistical test methods depend on 10 or more samples per phase; with 20 or more being even better. The following is a recommended procedure to obtain a successful statistical approach for validating proposed PCC reduction/termination activities:

1. Plot the data to view the fluctuations in the data with time. Does it appear that trends exist? Are there possible seasonal or cyclic trends to the data?

Note 15 – There is a strong possibility that a seasonal effect could occur with the emission data since temperature and precipitation affect the biological activity which produces the desired MSW stabilization. References 3 to 5 discuss the Seasonal Kendall (or Mann-Kendall) test for seasonality effects on trends.

2. Determine if there are distinct data phases based on their variations and known operational situations.

Note 16 – All of the stability parameters have a relationship with each other; some more direct than others. These relationships are affected by landfill operational factors; hence, changes in the latter should help to identify potential phase changes.

3. Examine the data to see if there are obvious situations and/or errors which could explain outliers or abrupt changes in the data trends.

Note 17 - Obvious situations could include a change in phase operation, a change in a laboratory procedure for a given analyte or a slug input which caused the emission values to change. Obvious errors could include data which are not in agreement with each other, e.g., COD data which are less than the BOD data or ammonia data which are expressed as nitrogen data when it is really expressed as ammonia as a molecule; or vice versa.

4. Are there any non-detect (or censored) data even thought the Kendall's tau method doesn't not require any special care if such data exist and the typical stabilization parameters are not non-detectable.

Note 18 – They are statistical methods (see References 3 to 5) which can be used to compensate for this kind of data. EPA has provided an Excel 2010 groundwater (see **Note 19**) statistics tool which determines parametric and non-parametric statistics including the Mann-Kendall statistic (12). The various statistical tests are based on Reference 4 and the tool is designed to support a companion guidance document (13).

Note 19 – A parallel situation can be drawn between groundwater (GW) remediation and MSW stabilization in that both processes approach equilibrium. GW cleanup results in contaminant concentrations which approach equilibriums which are less than the MCLs and MSW contaminant levels reach equilibrium as the stabilization processes are completed.

5. Using available software, determine if the available analytical data are normally distributed. Again, this is not necessary since Kendall's tau does not depend on whether the data are normally or not normally distributed. Regardless, an inspection of the distributions will give insight into the relative nature of the different parameters.

6. Identify the three five plus year phases (considering Item 2 phases) that can be used to determine if equilibrium has been achieved for the selected stabilization parameters as per the policy and guidance documents mentioned above.

7. Compute Kendall's tau to validate conclusions derived from other available landfill data.

List of References

1. **Reduction and/or Termination of Postclosure Care Activities**, BWM Policy 2014-P2 (5-1-14).
2. **Preparation of Postclosure Care Reduction and/or Termination Plans**, BWM Technical Guidance Document SW-2014-G1 (5-2-14).
3. **Statistical Methods in Water Resources** by D. R. Helsel and R. M. Hirsch, Chapter A2 of Book 4, Hydrologic Analysis and Interpretation of Techniques of Water-Resources Investigations of the United States Geological Survey (September 2002). 544 pages
4. **Statistical Analysis of Groundwater Monitoring Data at RCRA Facilities**, Unified Guidance, EPA 530/$-09-007 (March 2009). 888 pages

5. **Groundwater Statistics and Monitoring Compliance: Statistical Tools for the Project Life Cycle**, Interstate Technology & Regulatory Council and Groundwater Statistics and Monitoring Compliance Team (December 2013). 371 pages
6. **Statistics Tutorial**, see: www.gla.ac.uk/sums/users/jdbmcdonald/PrePost_TTest/instruct1.html
7. **Practical Nonparametric Statistics** by W. J. Conover, 3rd Edition, Wiley-India (1999).
8. **Nondetects and data analysis: statistics for censored environmental data** by D. R. Helsel, John Wiley & Sons, New Jersey (2005).
9. **Minitab**, see: http://www.minitab.com/en-US/default.aspx
10. **Landfill Gas Sampling Plan for Reduction and/or Termination of Post-closure Care (SW-2014-G2).**
11. **Leachate Sampling Plan for Reduction/Termination of Postclosure Care (SW-2013-G3).**
12. **Groundwater Statistics Tool: User's Guide**, Office of Solids Waste and Emergency Response, Office of Superfund Remediation and Technology Innovation (July 2014).
13. **The Guidance for Evaluating Completion of Groundwater Restoration Remedial Actions**, OSWER 9355.0-129 (2013).

Acknowledgment

Thanks are extended to Mike Johnson of MLJ-LLC Ecosystems Consulting for his review and comments of the draft version of this document. Information about this firm is available from: http://mlj-llc.com/ .